RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.SC. FIRST SEMESTER (July – December), 2011

Mid-Semester Examination, September, 2011

Date : 12/09/2011

MATHEMATICS (Honours)

Time : 11 am – 1 pm

Paper : I

Full Marks : 50

(Use Separate answer scripts for each group)

Group - A

1. Answer **any three** questions : [3×5 = 15]

   a) Let $Q' = Q - \{1\}$. Prove that Q' is an abelian group with respect to '$*$' defined by $a * b = a + b - ab$; $a, b \in Q'$

   b) Let H, K be two subgroups of a group G. Prove that $H \cup K$ is a subgroup of G iff either $H \subseteq K$ or $K \subseteq H$.

   c) Let G be an open set in $\mathbb{R}$. If G forms a subgroup of ($\mathbb{R}$, +), prove that G = $\mathbb{R}$.

   d) Let p and q be two positive integers such that gcd(p,q) = 1 and let $G = \left\langle \frac{p}{q} \right\rangle$, the cyclic subgroup of ($\mathbb{R}$, +) generated by $\frac{p}{q}$. If $Z + G = \{x + y : x \in Z, y \in G\}$, prove that Z+G is a cyclic group generated by $\frac{1}{q}$.

   e) Define the index of a subgroup in a group. Prove that $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$ is a subgroup of the multiplicative group $\mathbb{R}^*$ of all non-zero real numbers. Find the index of $\mathbb{R}^+$ in $\mathbb{R}^*$.

Group – B

2. Answer **any two** questions : [2×5 = 10]

   a) i) State and prove well ordering property of $\mathbb{N}$

      ii) Let S be a non empty subset of $\mathbb{R}$, bounded above and $T = \{-x;\ x \in S\}$. Prove that T is bounded below and inf T = – sup S. [(1+2)+2]

   b) i) Let $x \in \mathbb{R}$ and x > 0. Show that there exists a natural number m such that $m - 1 \le x < m$.

      ii) Prove that between any two real numbers there exists an irrational number. [2+3]

   c) Prove that union of an arbitrary collection of open sets in $\mathbb{R}$ is also an open set. Is the result true for arbitrary intersection? Justify your answer. [3+2]

   d) i) Let $S \subset \mathbb{R}$ and c be a limit point of S. Prove that every neighboured of c contains infinitely many elements of S.

      ii) Give an example of an infinite set $S \subset \mathbb{R}$ such that S has exactly three limit points. [3+2]

Group – C

Answer **any two** questions : [2×5 = 10]

3. a) Show that the equation of the line joining the feet of perpendiculars from the point (d, 0) on the lines $ax^2 + 2hxy + by^2 = 0$ is $(a - b)x + 2hy + bd = 0$ [5]

b) Prove that the orthocentre (p,q) of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $\ell x + my = 1$ is given by $\frac{p}{\ell} = \frac{q}{m} = \frac{a+b}{am^2 - 2h\ell m + b\ell^2}$ [5]

c) Show that the line $r\cos(\theta - \alpha) = p$ touches the conic $\frac{\ell}{r} = 1 + e\cos\theta$ if $(\ell\cos\alpha - ep)^2 + \ell^2\sin^2\alpha = p^2$ [5]

Group – D

Answer **Question No. 4** and **any one** from Q. No. 5 & Q. No. 6 :

4. Answer **any one** : [5]

a) Solve, using the method of variation of parameters, the equation

$$\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$$

b) Reduce the differential equation y = 2px – p^2y to Clairaut's form by the substitutions y^2 = Y, x = X and then obtain the complete primitive and singular solution, if any.

5. a) Solve the equation $(2x^2y - 3y^4)dx + (3x^3 + 2xy^3)dy = 0$.

b) Prove that $\frac{1}{(x+y+1)^4}$ is an integrating factor of $(2xy - y^2 - y)dx + (2xy - x^2 - x)dy = 0$. [5+5]

6. a) Find the general and singular solutions of $16x^2 + 2p^2y - p^3x = 0$.

b) Solve, by the method of variation of parameters, the differential equation $(1+x)\frac{dy}{dx} - xy = 1 - x$. [5+5]